Surface Recession and Backface Temperature of Laser-Irradiated Opaque Slabs

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Abstract

THE purpose of this work is to derive broadly applicable equations of immediate relevance for investigating continuous-wave/laser-radiation interactions with material targets that vaporize without first melting. The procedure is based on the heat-balance integral method and emphasizes the use of nondimensional variables. Specifically, the author demonstrates how to solve the differential equations that describe the front-surface recession and the backface-temperature evolution in a burnthrough mode.

Contents

In the framework of a one-dimensional model, and prior to the onset of vaporization, the thermal response of a laser-irradiated solid slab having temperature-independent properties and an insulated backface can be obtained by various analytical methods.[1] For our purposes, we find it convenient to introduce dimensionless variables for the temperature, the position, and the time: $\Theta = T/T_V$, $\zeta = z/l$, and $\tau = t/t_d$, where T_V is the vaporization temperature, l is the thickness of the slab, and t_d is the nominal diffusion time, i.e.,

$$t_d = l^2/(\pi^2\kappa) \tag{1}$$

This allows us to express the evolution of the laser-induced temperature rise in a compact manner

$$\Theta(\zeta,\tau) = F\left\{\frac{\tau}{\pi^2} + \frac{1}{2}(1-\zeta)^2 - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\exp(-n^2\tau)\cos[n\pi(1-\zeta)]\right\} \tag{2}$$

if the coupled irradiance αI is in units of KT_V/l,

$$F = \alpha I l/(KT_V) \tag{3}$$

K referring to the thermal conductivity. Front-surface vaporization occurs at a time τ_V for which $\Theta(0, \tau_V) = 1$, or

$$\tau_V + \frac{\pi^2}{3} - 2\sum_{n=1}^{\infty}\frac{\exp(-n^2\tau_V)}{n^2} - \frac{\pi^2}{F} = 0 \tag{4}$$

which yields

$$\tau_V \simeq \pi^2(1/F - 1/3) \qquad \tau_V \geq 3 \tag{5a}$$

$$\tau_V \simeq 7.72/F^2 \qquad \tau_V < 3 \tag{5b}$$

for long and short exposures, respectively. Regarding backface temperatures, Eq. (2) indicates that we have

$$\Theta(1, \tau_V) = F\left[\frac{\tau_V}{\pi^2} - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\exp(-n^2\tau_V)\right] \tag{6}$$

at the onset of vaporization, and thus the backface temperature cannot exceed a level of $1 - F/2$ in the case of long exposures.

In the context of an application of the heat-balance integral (HBI) method as implemented by Harrach,[2] the temperature profile at the onset of vaporization is described by

$$\Theta(\zeta, \tau_V) = (1 - \zeta/\delta_V)^2 \exp(-\zeta/\delta_V) \tag{7}$$

for $\zeta \leq \delta_V$, δ_V representing the depth of thermal penetration. The boundary condition at the front surface, i.e.,

$$-\left.\frac{\mathrm{d}\Theta(\zeta,\tau_V)}{\mathrm{d}\zeta}\right|_{\zeta=0} = F \tag{8}$$

then tells us that the relation

$$\delta_V = 3/F \tag{9}$$

holds, and hence suggests that ablation without backface heating requires $F > 3$ (high intensities), whereas $F \leq 3$ (low intensities) implies that the entire ablation proceeds under conditions that involves backface heating. In the next paragraph we will first consider high-intensity-type situations in the absence of any backface heating.

For this purpose, we introduce three dimensionless parameters: $\Delta = \delta/\delta_V - 1$, $\eta = H_V/(C_pT_V)$, and $\theta = \tau/\tau_V$ to characterize the thermal penetration, the vaporization enthalpy, and the elapsed time, respectively. The HBI method then indicates that the front-surface recession rate can be expressed as follows:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}[\zeta_s(\tau)] = \frac{F}{\pi^2\eta}\cdot\frac{\Delta(\theta)}{1+\Delta(\theta)} \tag{10}$$

where $\Delta(\theta)$ is a solution of

$$\left[1 - \frac{\Delta(\theta)}{\eta}\right]\exp\left[\frac{\Delta(\theta)}{1+\eta}\right] - \exp\left[-\frac{\theta-1}{\eta(1+\eta)}\right] = 0 \tag{11}$$

which cannot be generated in closed form. For $\theta \gg 1$ we have $\Delta \to \eta$, which demonstrates that in the absence of backface heating the recession rate tends toward a steady-state limit

$$v_{SS} = F/[\pi^2(1+\eta)] \tag{12}$$

that relates directly to the burnthrough time. The surface recession depth, that is,

$$\zeta_s(\tau_0) = \int_{\tau_V}^{\tau_0}\frac{\mathrm{d}}{\mathrm{d}\tau}[\zeta_s(\tau)]\,\mathrm{d}\tau \tag{13}$$

if τ_0 measures the duration of the exposure in units of t_d, thus amounts to

$$\zeta_s(\theta_0) = \frac{F\tau_V}{\pi^2\eta}\int_1^{\theta_0}\frac{\Delta(\theta)}{1+\Delta(\theta)}\,\mathrm{d}\theta \tag{14}$$

and can be directly evaluated to describe the evolution of the crater depth, particularly during the transient phase. At this point, we re-emphasize that Eq. (14) applies only if the exposure time is smaller than or equal to an effective diffusion time in the sense of our model; in effect, the diffusion time τ_D

Presented as Paper 84-1782 at the AIAA 19th Thermophysics Conference, Snowmass, CO, June 25–28, 1984; synoptic received May 27, 1986. Full paper available from AIAA Library, 555 W. 57th St., New York, NY 10019. Price: microfiche, $4.00, hard copy, $9.00. **Remittance must accompany order.**

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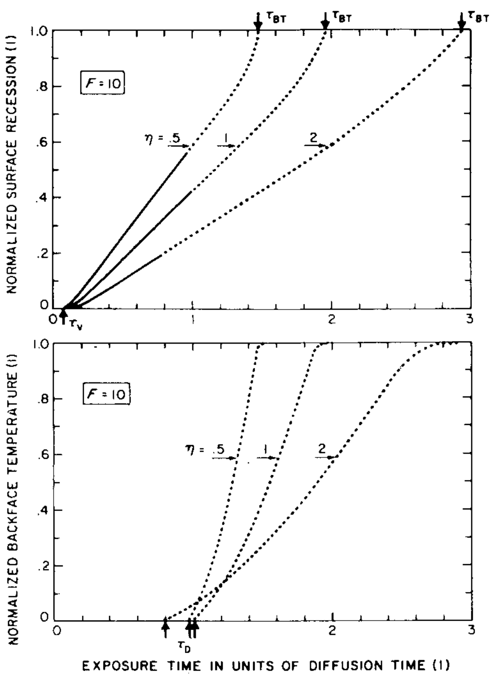

Fig. 1 Surface recession and backface temperature in the high-intensity regime.

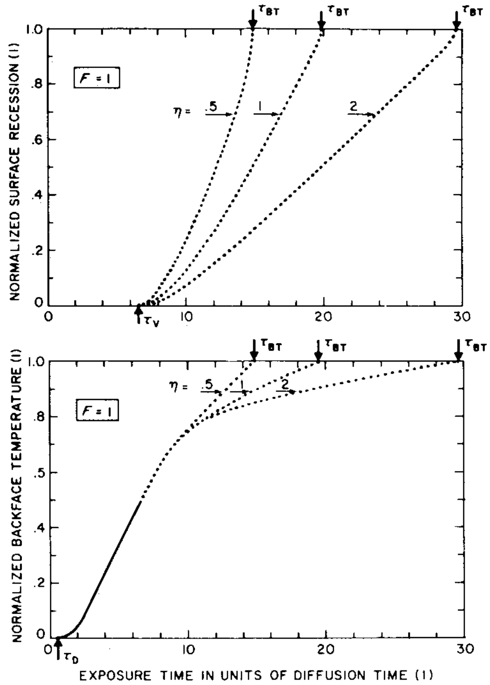

Fig. 2 Surface recession and backface temperature in the low-intensity regime.

is a function of beam intensity and target characteristics that solves the integral equation

$$\zeta_s(\theta_D)+[\Delta(\theta_D)+1]\delta_V=1 \tag{15}$$

The solution requires numerical techniques.

Under conditions such that the backface of the irradiated slab experiences appreciable heating ultimately resulting in burnthrough, two distinct cases must be considered depending upon the magnitude of the dimensionless irradiance F. At high intensities ($F>3$), Harrach's *Ansatz*[2] leads to an ordinary differential equation for the front-surface position

$$\frac{\mathrm{d}}{\mathrm{d}\tau}[\zeta_s(\tau)]=\frac{3}{\pi^2\eta}\left[\frac{F}{3}-\frac{1-\Theta(1,\tau)}{1-\zeta_s(\tau)}\right] \tag{16a}$$

if it is understood that $\Theta(1,\tau)$ relates to $\zeta_s(\tau)$ by way of

$$1-\Theta(1,\tau)=$$

$$\frac{1-\zeta_s(\tau_D)+(e\eta/2)[\zeta_s(\tau)-\zeta_s(\tau_D)]-[eF/(2\pi^2)](\tau-\tau_D)}{1-\zeta_s(\tau)} \tag{16b}$$

where τ_D and $\zeta_s(\tau_D)$ are initial values as given by Eqs. (15) and (14), respectively. For $F=10$ and η ranging from 0.5 to 2, the Runge-Kutta procedure then yields the solutions illustrated in Fig. 1 (dotted segments). At low intensities ($F\leq 3$), and in the framework of the methodology proposed here, the system in Eqs. (16a) and (16b) must be rewritten as follows:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}[\zeta_s(\tau)]=\frac{3}{\pi^2\eta}\left[1-\Theta(1,\tau_V)-\frac{1-\Theta(1,\tau)}{1-\zeta_s(\tau)}\right] \tag{17a}$$

$$1-\Theta(1,\tau)=$$

$$\frac{1-\Theta(1,\tau_V)+(e\eta/2)\zeta_s(\tau)-[eF/(2\pi^2)](\tau-\tau_V)}{1-\zeta_s(\tau)} \tag{17b}$$

with initial values τ_V and $\Theta(1,\tau_V)$ obtained from Eqs. (5) and (6). An application for the case of $F=1$ is displayed in Fig. 2

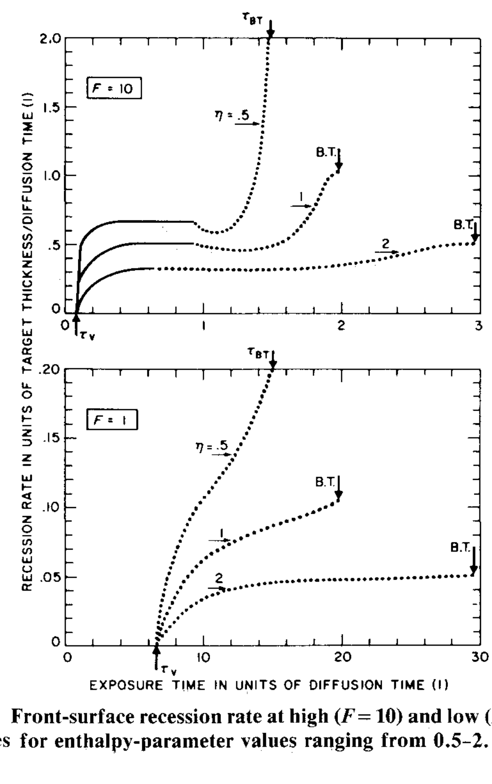

Fig. 3 Front-surface recession rate at high ($F=10$) and low ($F=1$) intensities for enthalpy-parameter values ranging from 0.5–2.

and is seen to yield results that are consistent with the preablation evolution of the backface temperature. Since surface position and backface temperature are now both available as a function of elapsed time, it becomes a straightforward matter to derive the recession rate simply through substitution into Eq. (16a) or (17a). Figure 3 shows recession-rate histories that were generated in this manner, that is, on using the data plotted in Figs. 1 and 2.

References

[1]Carslaw, H.S. and Jaeger, J.C., *Conduction of Heat in Solids*, Oxford University Press, London, UK, 1959.

[2]Harrach, R.J., "Analytical Solutions for Laser Heating and Burnthrough of Opaque Solid Slabs," *Journal of Applied Physics*, Vol. 48, June 1977, p. 2370–2389.